AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS

Unit Pure Core 3

Wednesday 14 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $y = (\sin 4x)(\sec 3x)$, use the product rule to find $\dfrac{dy}{dx}$.

[2 marks]

(b) Find $\displaystyle\int \frac{6x}{2x^2 + 3}\,dx$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	Answer space for question 1

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2 (a) Use the mid-ordinate rule with five strips to find an estimate for $\int_{0.5}^{1.5} e^{3x-x^3}\, dx$,

giving your answer to three decimal places.

[4 marks]

(b) A curve has equation $y = e^{3x-x^3}$. Find the exact values of the coordinates of the stationary points of the curve and determine the nature of these stationary points.

[7 marks]

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3 Use the substitution $u = \cos 2x$ to find

$$\int \cos^2 2x \sin^3 2x \, dx$$

[5 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE	Answer space for question 3

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4 The line $y = x$ and the curve with equation $y = \ln\left(\frac{3x+10}{3x+1}\right)$, where $x > 0$, intersect at a single point where $x = \alpha$.

(a) Show that α lies between 1 and 2.

[2 marks]

(b) (i) Use the iterative formula

$$x_{n+1} = \ln\left(\frac{3x_n+10}{3x_n+1}\right)$$

with $x_1 = 2$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(ii) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = \ln\left(\frac{3x+10}{3x+1}\right)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

[2 marks]

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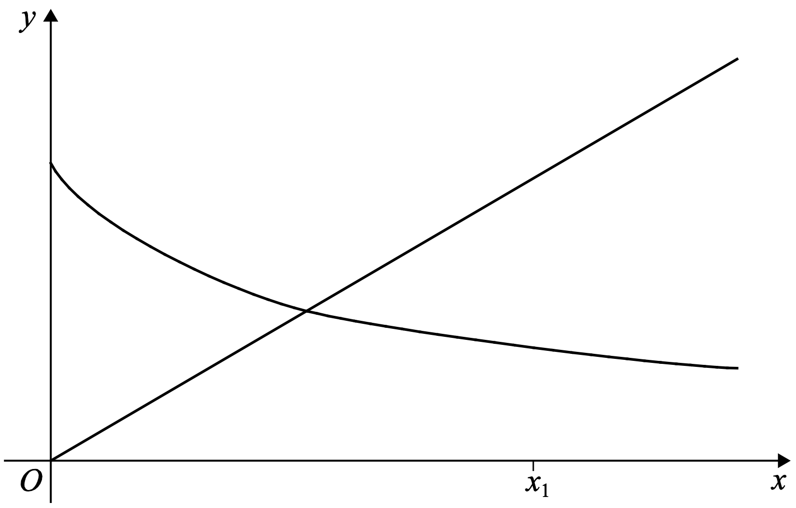
Figure 1



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5 The function f is defined by

$$f(x) = \ln(3x + 1), \text{ for } x \geqslant 0$$

The function g is defined by

$$g(x) = \frac{d}{dx}(f(x)), \text{ for } x \geqslant 0$$

The inverse of f is f^{-1}.

(a) Find expressions for $f^{-1}(x)$ and $g(x)$.

[4 marks]

(b) Show that the equation $f^{-1}(x) = g(x)$ can be rearranged into the form

$$x = \ln\left(\frac{3x + 10}{3x + 1}\right)$$

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 Use integration by parts to find the value of $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx$.

[6 marks]

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7 You are given that k is a positive constant.

By sketching the graphs of $y = |5x - 3k|$ and $y = 3|x + 4k|$ on the same axes, solve the inequality

$$|5x - 3k| \geqslant 3|x + 4k|$$

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

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8 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2\left(2x-\frac{\pi}{6}\right)=11-\sec\left(2x-\frac{\pi}{6}\right)$$

giving all values of x in radians to two decimal places in the interval $0 \leqslant x \leqslant \pi$.

[7 marks]

(b) Describe a sequence of **two** geometrical transformations that maps the graph of $y=\mathrm{f}\left(2x-\frac{\pi}{6}\right)$ onto the graph of $y=\mathrm{f}(x)$.

[4 marks]

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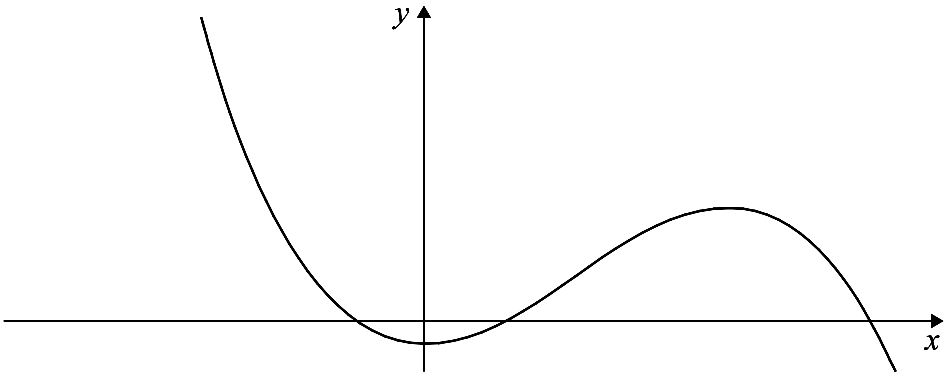
9 **Figure 2** shows part of the curve with equation $y = \mathrm{f}(x)$.

Figure 2



(a) On **Figure 3** below, sketch the curve with equation $y = |\mathrm{f}(x)|$.

[3 marks]

(b) On **Figure 4** opposite, sketch the curve with equation $y = -\mathrm{f}(|x|)$.

[2 marks]

(c) The curve with equation $y = \mathrm{f}(x)$ has a minimum point at $(0, b-2)$ and a maximum point at $(a, 9b)$, where $0 < b < 2$.

(i) Find the coordinates of the minimum point of the curve with equation $y = \mathrm{f}(x+a) + 2b$.

[2 marks]

(ii) Find the coordinates of the maximum point of the curve with equation $y = 3\mathrm{f}(2x)$.

[2 marks]

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Figure 3

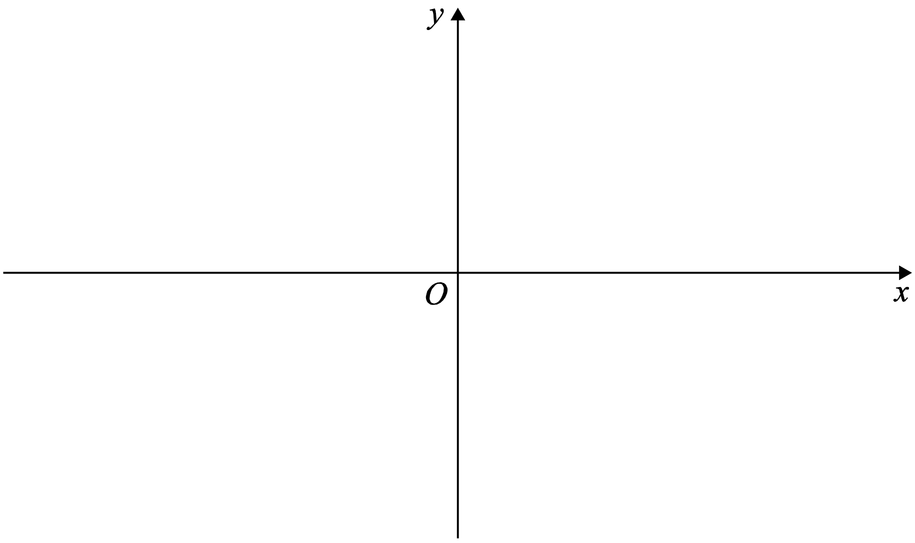

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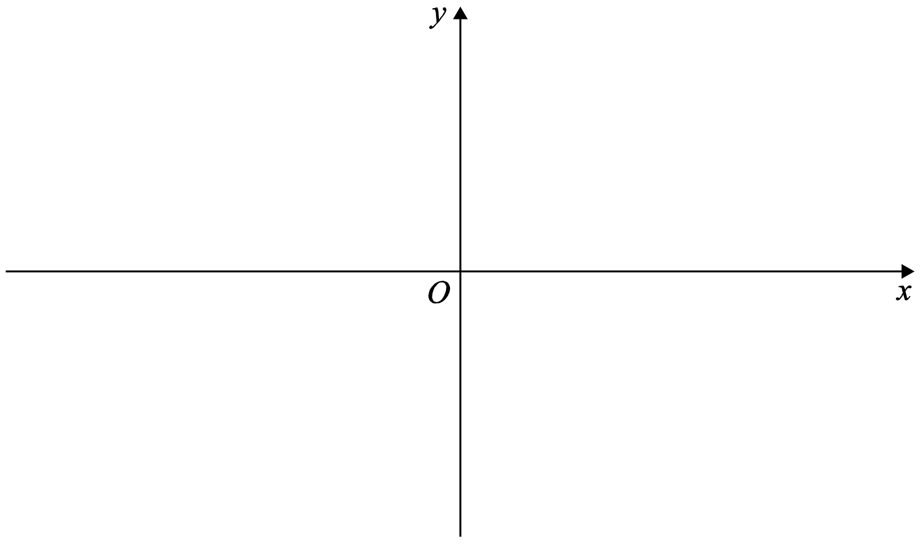
Figure 4



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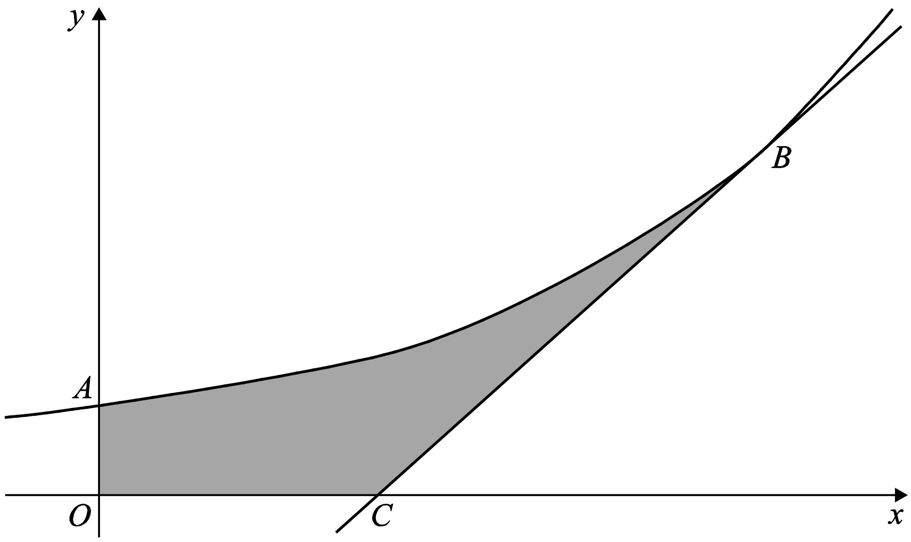
10 The diagram shows the curve $y = e^{2x}$, intersecting the y-axis at the point A, and the tangent to this curve at the point B, where $x = \ln 4$, intersecting the x-axis at the point C.



(a) (i) Find an equation of the tangent to the curve at B.

[3 marks]

(ii) Hence show that the coordinates of C are $\left(\ln 4 - \frac{1}{2},\ 0\right)$.

[1 mark]

(b) The shaded region $OABC$ is rotated through 2π radians about the x-axis to form a solid.

Find the **exact** value of the volume of the solid generated.

(You may assume that the volume of a cone of radius r and height h is $\frac{1}{3}\pi r^2 h$.)

[8 marks]

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